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7.1 Continuous Random Variables: Introduction TCC
This module serves as an introduction to the Continuous Random Variables chapter in the Elementary Statistics textbook.

Student Learning Objectives

By the end of this chapter, the student should be able to:

- Recognize and understand continuous probability density functions in general.
- Recognize the uniform probability distribution and apply it appropriately.

Introduction

Continuous random variables have many applications. Baseball batting averages, IQ scores, the length of time a long distance telephone call lasts, the amount of money a person carries, the length of time a computer chip lasts, and SAT scores are just a few. The field of reliability depends on a variety of continuous random variables.

This chapter gives an introduction to continuous random variables and the many continuous distributions. We will be studying these continuous distributions for several chapters.

The characteristics of continuous random variables are:

- The outcomes are measured, not counted.
- Geometrically, the probability of an outcome is equal to an area under a mathematical curve called the density curve, $f(x)$.
- Each individual value has zero probability of occurring. Instead we find the probability that the value is between two endpoints.

We will start with a simplest continuous distributions, the **Uniform**.

Note: The values of discrete and continuous random variables can be ambiguous. For example, if X is equal to the number of miles (to the nearest mile) you drive to work, then X is a discrete random variable. You count the miles. If X is the distance you drive to work, then you measure values of X and X is a continuous random variable. How the random variable is defined is very important.

Glossary

Uniform Distribution

A continuous random variable (RV) that has equally likely outcomes over the domain, $a < x < b$. Often referred as the **Rectangular distribution** because the graph of the pdf has the form of a rectangle. Notation: $X\text{~}U(a,b)$. The mean is $\mu = \frac{a+b}{2}$ and the standard deviation is $\sigma = \sqrt{\frac{(b-a)^2}{12}}$ The probability density function is $f(X) = \frac{1}{b-a}$ for $a < X < b$ or $a \leq X \leq b$. The cumulative distribution is $P(X \leq x) = \frac{x-a}{b-a}$.

Exponential Distribution

A continuous random variable (RV) that appears when we are interested in the intervals of time between some random events, for example, the length of time between emergency arrivals at a hospital. Notation: $X\text{~}\mathrm{Exp}(m)$. The mean is $\mu = \frac{1}{m}$ and the standard deviation is $\sigma = \frac{1}{m}$. The probability density function is $f(x) = \mathrm{me}^{-\mathrm{mx}}$, $x \geq 0$ and the cumulative distribution function is $P(X \leq x) = 1 - e^{-\mathrm{mx}}$.

7.2 Continuous Random Variables: Continuous Probability Functions Uniform Distributions Part 1 TCC

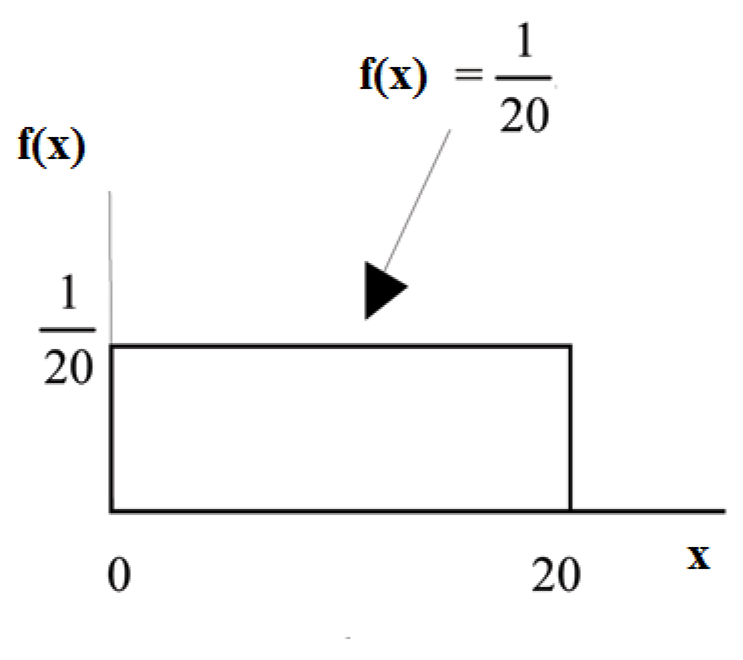
This module introduces the continuous probability function and explores the relationship between the probability of X and the area under the curve of f(X).

We begin by defining a continuous probability density function. We use the function notation $f(x)$. Intermediate algebra may have been your first formal introduction to functions. In the study of probability, the functions we study are special. We define the function $f(x)$ so that the area between it and the x-axis is equal to a probability. Since the maximum probability is one, the maximum area is also one.

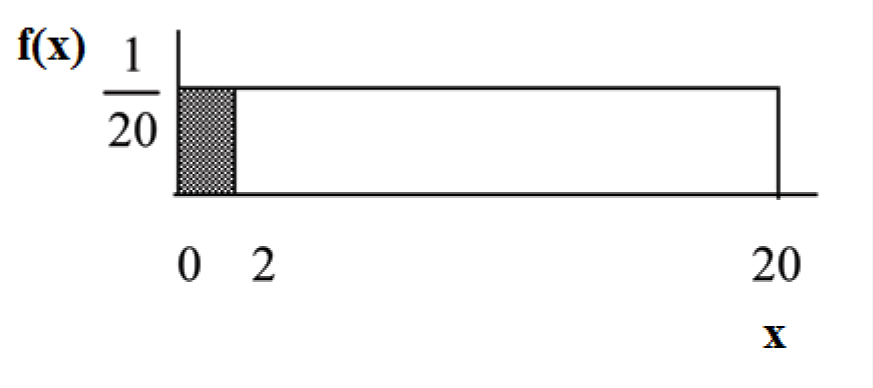
For continuous probability distributions, PROBABILITY = AREA.

Example:

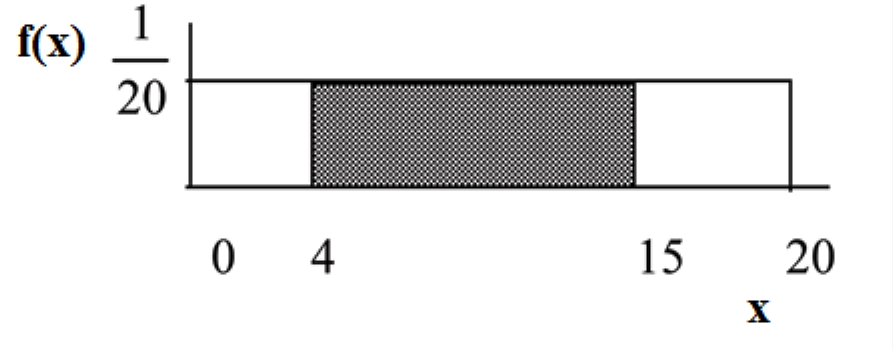
Consider the function $f(x) = \frac{1}{20}$ for $0 \leq x \leq 20$. x = a real number. The graph of $f(x) = \frac{1}{20}$ is a horizontal line. However, since $0 \leq x \leq 20$, $f(x)$ is restricted to the portion between $x = 0$ and $x = 20$, inclusive .



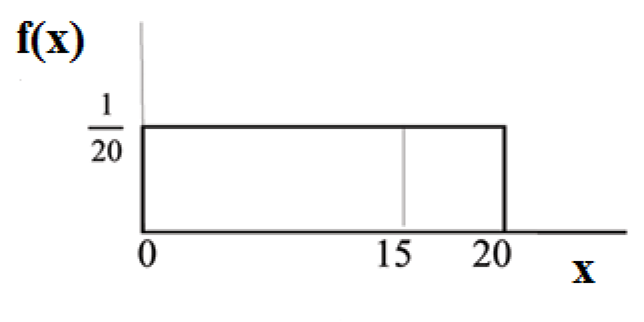
$f(x) = \frac{1}{20}$ **for** $0 \leq x \leq 20$.
The graph of $f(x) = \frac{1}{20}$ is a horizontal line segment when $0 \leq x \leq 20$.
The area between $f(x) = \frac{1}{20}$ where $0 \leq x \leq 20$ and the x-axis is the area of a rectangle with base = 20 and height $= \frac{1}{20}$.
$\text{AREA} = 20 \cdot \frac{1}{20} = 1$
This particular function, where we have restricted x so that the area between the function and the x-axis is 1, is an example of a continuous probability density function. It is used as a tool to calculate probabilities.
Suppose we want to find the area between $f(x) = \frac{1}{20}$ **and the x-axis where** $0 < x < 2$ **.**



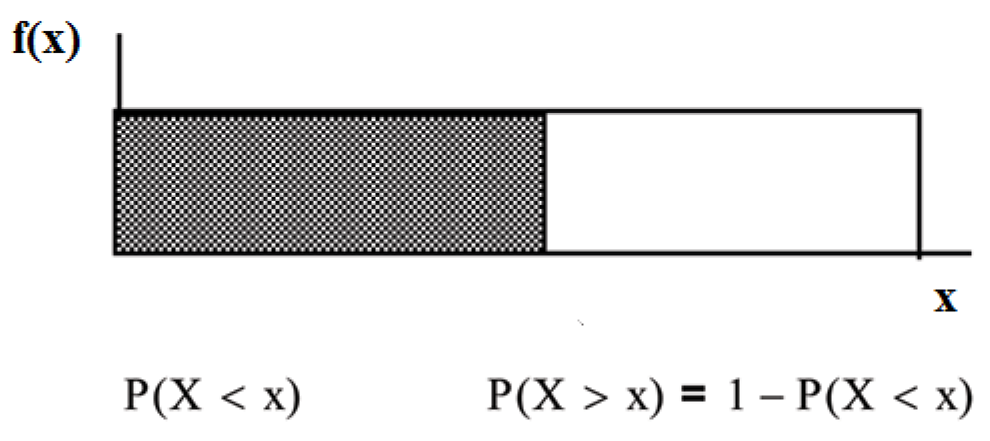
$\text{AREA} = (2 - 0) \cdot \frac{1}{20} = 0.1$
$(2 - 0) = 2 =$ base of a rectangle
$\frac{1}{20}$ = the height.
The area corresponds to a probability. The probability that x is between 0 and 2 is 0.1, which can be written mathematically as
$P(0<x<2) = P(x<2) = 0.1$.
Suppose we want to find the area between $f(x) = \frac{1}{20}$ **and the x-axis where** $4 < x < 15$ **.**



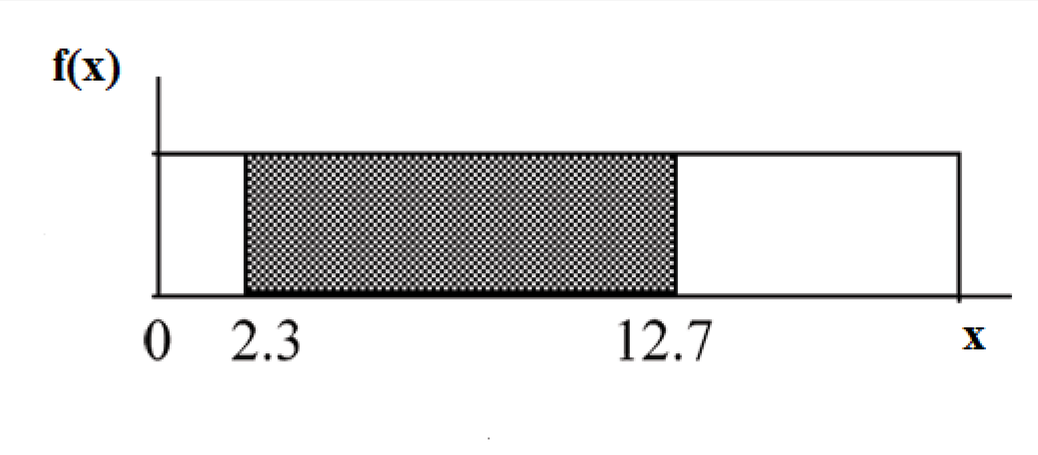
$\text{AREA} = \ 15 - 4 \ \cdot \frac{1}{20} = 0.55$
$(15 - 4) = 11 = \text{the base of a rectangle}$
$\frac{1}{20}$ = the height.
The area corresponds to the probability $P(4 < x < 15) = 0.55$.
Suppose we want to find $P(x{=}15)$**.** On an x-y graph, $x{=}15$ is a vertical line. A vertical line has no width (or 0 width). Therefore,
$P(x{=}15) = (\text{base})(\text{height}) = \ 0 \ \ \frac{1}{20} \ \ = 0.$



$P(X \leq x)$ (can be written as $P(X < x)$ for continuous distributions) is called the cumulative distribution function or CDF. Notice the "less than or equal to" symbol. We can use the CDF to calculate $P(X > x)$. The CDF gives "area to the left" and $P(X > x)$ gives "area to the right." We calculate $P(X > x)$ for continuous distributions as follows:
$P(X > x) = 1 - P(X < x)$.



Label the graph with f(x) and x. Scale the x and y axes with the maximum x and y values. $f(x) = \frac{1}{20}$, $0 \leq x \leq 20$.



$P(2.3 < x < 12.7) = \text{(base)(height)} = \quad 12.7 - 2.3 \quad \frac{1}{20} \quad = 0.52$

7.3 Continuous Random Variables: Continuous Probability Functions Uniform Distributions Part 2 TCC Continuous Random Variable: Uniform Distribution is part of the collection col10555 written by Barbara Illowsky and Susan Dean. It describes the properties of the Uniform Distribution with contributions from Roberta Bloom.

Example:
The previous problem is an example of the **uniform probability distribution**.
Illustrate the uniform distribution. The data that follows are 55 smiling times, in seconds, of an eight-week old baby.

10.4	19.6	18.8	13.9	17.8	16.8	21.6	17.9	12.5	11.1	4.9
12.8	14.8	22.8	20.0	15.9	16.3	13.4	17.1	14.5	19.0	22.8
1.3	0.7	8.9	11.9	10.9	7.3	5.9	3.7	17.9	19.2	9.8
5.8	6.9	2.6	5.8	21.7	11.8	3.4	2.1	4.5	6.3	10.7
8.9	9.4	9.4	7.6	10.0	3.3	6.7	7.8	11.6	13.8	18.6

sample mean = 11.49 and sample standard deviation = 6.23
We will assume that the smiling times, in seconds, follow a uniform distribution between 0 and 23 seconds, inclusive. This means that any smiling time from 0 to and including 23 seconds is **equally likely**. The histogram that could be constructed from the sample is an empirical distribution that closely matches the theoretical uniform distribution.
Let X = length, in seconds, of an eight-week old baby's smile.
The notation for the uniform distribution is
$X \sim U(\mathrm{a}, b)$ where a = the lowest value of x and b = the highest value of x.
The probability density function is $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$.
For this example, $x \sim U(0,23)$ and $f(x) = \frac{1}{23-0}$ for $0 \leq x \leq 23$.
Formulas for the theoretical mean and standard deviation are
$\mu = \frac{a+b}{2}$ and $\sigma = \sqrt{\frac{(b-a)^2}{12}}$
For this problem, the theoretical mean and standard deviation are
$\mu = \frac{0+23}{2} = 11.50$ seconds and $\sigma = \sqrt{\frac{(23-0)^2}{12}} = 6.64$ seconds
Notice that the theoretical mean and standard deviation are close to the sample mean and standard deviation.

Example:
Exercise:

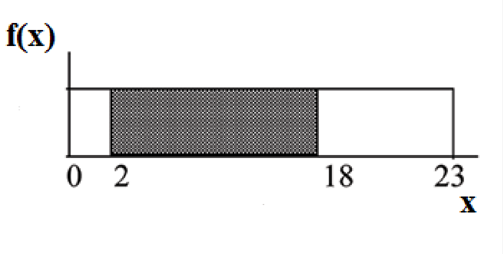
Problem:

What is the probability that a randomly chosen eight-week old baby smiles between 2 and 18 seconds?

Solution:

Find $P(2 < x < 18)$.

$P(2 < x < 18) = (\text{base})(\text{height}) = \left(18 - 2\right) \cdot \frac{1}{23} = \frac{16}{23}$.



Exercise:

Problem: Find the 90th percentile for an eight week old baby's smiling time.

Solution:

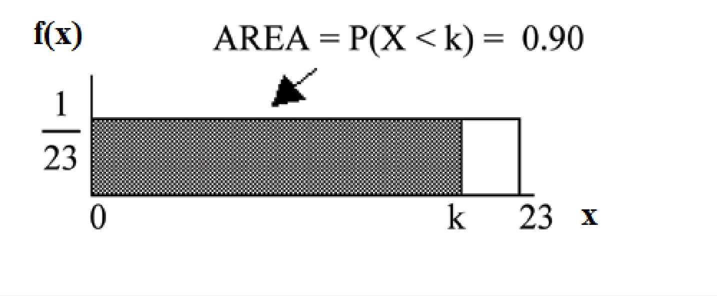
Ninety percent of the smiling times fall below the 90th percentile, k, so $P(x < k) = 0.90$

$P(x < k) = 0.90$

$(\text{base})(\text{height}) = 0.90$

$\left(k - 0\right) \cdot \frac{1}{23} = 0.90$

$k = 23 \cdot 0.90 = 20.7$



Exercise:

Problem:

Find the probability that a random eight week old baby smiles more than 12 seconds **KNOWING** that the baby smiles **MORE THAN 8 SECONDS**.

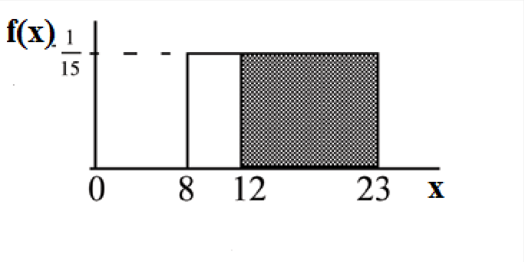
Solution:

Find $P(x > 12 | x > 8)$ There are two ways to do the problem. **For the first way**, use the fact that this is a **conditional** and changes the sample space. The graph illustrates the new sample space. You already know the baby smiled more than 8 seconds.

Write a new $f(x)$: $f(x) = \frac{1}{23-8} = \frac{1}{15}$

for $8 < x < 23$

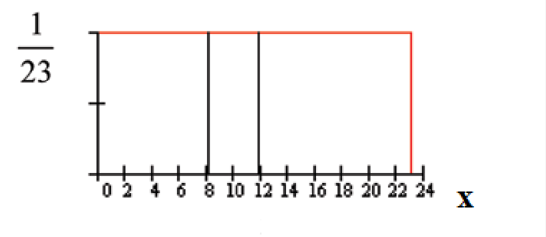
$P(x > 12 | x > 8) = (23 - 12) \cdot \frac{1}{15} = \frac{11}{15}$



For the second way, use the conditional formula from **Probability Topics** with the original distribution $X \sim U(0, 23)$:

$P\left(A \middle| B\right) = \frac{P(A \text{ AND } B)}{P(B)}$ For this problem, A is $(x > 12)$ and B is $(x > 8)$.

So, $P(x > 12 | x > 8) = \frac{(x>12 \text{ AND } x>8)}{P(x>8)} = \frac{P(x>12)}{P(x>8)} = \frac{\frac{11}{23}}{\frac{15}{23}} = 0.733$



Example:
Uniform: The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between 0 and 15 minutes, inclusive.
Exercise:

Problem: What is the probability that a person waits fewer than 12.5 minutes?

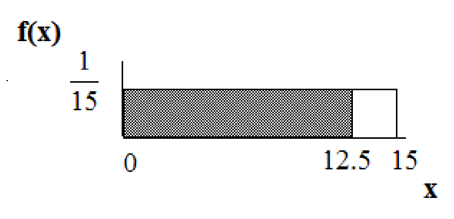
Solution:

Let X = the number of minutes a person must wait for a bus. a = 0 and b = 15. x~$U(0,15)$. Write the probability density function. $f(x)=\frac{1}{15-0}=\frac{1}{15}$ for 0≤ x ≤15.

Find $P(x<12.5)$. Draw a graph.

$$P(x<k)=(\text{base})(\text{height})=\left(12.5-0\right)\cdot\frac{1}{15}=0.8333$$

The probability a person waits less than 12.5 minutes is 0.8333.



Exercise:

Problem: On the average, how long must a person wait?

Find the mean, μ, and the standard deviation, σ.

Solution:

$\mu=\frac{a+b}{2}=\frac{15+0}{2}=7.5$. On the average, a person must wait 7.5 minutes.

$\sigma=\sqrt{\frac{(b-a)^2}{12}}=\sqrt{\frac{(15-0)^2}{12}}=4.3$. The Standard deviation is 4.3 minutes.

Exercise:

Problem: Ninety percent of the time, the time a person must wait falls below what value?

Note: This asks for the 90th percentile.

Solution:

Find the 90th percentile. Draw a graph. Let k = the 90th percentile.

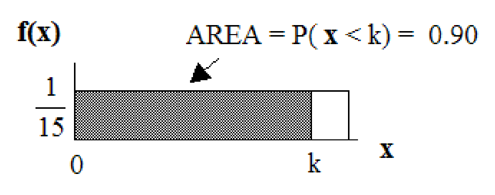
$$P(x<k)=(\text{base})(\text{height})=\left(k-0\right)\cdot\left(\frac{1}{15}\right)$$

$0.90 = k \cdot \frac{1}{15}$

$\mathrm{k} = (0.90)(15) = 13.5$

k is sometimes called a critical value.

The 90th percentile is 13.5 minutes. Ninety percent of the time, a person must wait at most 13.5 minutes.



Example:
Uniform: Suppose the time it takes a nine-year old to eat a donut is between 0.5 and 4 minutes, inclusive. Let X = the time, in minutes, it takes a nine-year old child to eat a donut. Then $X \sim U(0.5, 4)$.
Exercise:

Problem:

The probability that a randomly selected nine-year old child eats a donut in at least two minutes is _______.

Solution:

0.5714

Exercise:

Problem:

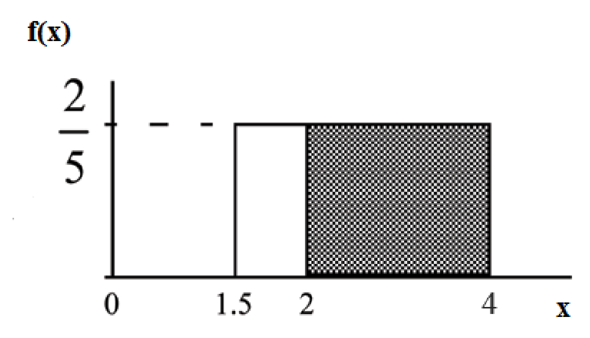
Find the probability that a different nine-year old child eats a donut in more than 2 minutes given that the child has already been eating the donut for more than 1.5 minutes.

The second probability question has a **conditional** (refer to "Probability Topics"). You are asked to find the probability that a nine-year old child eats a donut in more than 2 minutes given that the child has already been eating the donut for more than 1.5 minutes. Solve the problem two different ways (see the first example). You must reduce the sample space. **First way**: Since you already know the child has already been eating the donut for more than 1.5 minutes, you are no longer starting at $\mathrm{a} = 0.5$ minutes. Your starting point is 1.5 minutes.

Write a new f(x):

$f(x) = \frac{1}{4-1.5} = \frac{2}{5}$ for $1.5 \leq x \leq 4$.

Find $P(x > 2 | x > 1.5)$. Draw a graph.



$P(x > 2|x > 1.5) = (\text{base})(\text{new height}) = (4 - 2)(2/5) = ?$

Solution:

$\frac{4}{5}$

The probability that a nine-year old child eats a donut in more than 2 minutes given that the child has already been eating the donut for more than 1.5 minutes is $\frac{4}{5}$.

Second way: Draw the original graph for $x \sim U(0.5, 4)$. Use the conditional formula

$P(x > 2|x > 1.5) = \frac{P(x>2 \text{ AND } x>1.5)}{P(x>1.5)} = \frac{P(x>2)}{P(x>1.5)} = \frac{\frac{2}{3.5}}{\frac{2.5}{3.5}} = 0.8 = \frac{4}{5}$

Note: See "Summary of the Uniform and Exponential Probability Distributions" for a full summary.

Example:

Uniform: Ace Heating and Air Conditioning Service finds that the amount of time a repairman needs to fix a furnace is uniformly distributed between 1.5 and 4 hours. Let x = the time needed to fix a furnace. Then $x \sim U(1.5, 4)$.

1. Find the problem that a randomly selected furnace repair requires more than 2 hours.
2. Find the probability that a randomly selected furnace repair requires less than 3 hours.
3. Find the 30th percentile of furnace repair times.
4. The longest 25% of repair furnace repairs take at least how long? (In other words: Find the minimum time for the longest 25% of repair times.) What percentile does this represent?
5. Find the mean and standard deviation

Exercise:

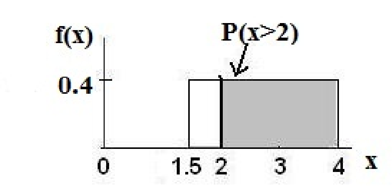
Problem: Find the probability that a randomly selected furnace repair requires longer than 2 hours.

Solution:

To find $f(x)$: $f(x) = \frac{1}{4-1.5} = \frac{1}{2.5}$ so $f(x)$ =0.4

P(x>2) = (base)(height) = (4 − 2)(0.4) = 0.8

Example 4 Figure 1



Uniform Distribution between 1.5 and 4 with shaded area between 2 and 4 representing the probability that the repair time x is greater than 2

Exercise:

Problem:

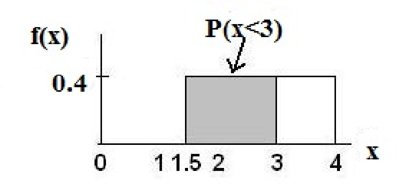
Find the probability that a randomly selected furnace repair requires less than 3 hours. Describe how the graph differs from the graph in the first part of this example.

Solution:

$P(x < 3)$ = (base)(height) = (3 − 1.5)(0.4) = 0.6

The graph of the rectangle showing the entire distribution would remain the same. However the graph should be shaded between x=1.5 and x=3. Note that the shaded area starts at x=1.5 rather than at x=0; since X~U(1.5,4), x can not be less than 1.5.

Example 4 Figure 2



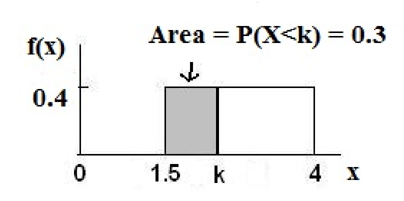
Uniform Distribution between 1.5 and 4 with shaded area between 1.5 and 3 representing the probability that the repair time x is less than 3

Exercise:

Problem: Find the 30th percentile of furnace repair times.

Solution:

Example 4 Figure 3



Uniform Distribution between 1.5 and 4 with an area of 0.30 shaded to the left, representing the shortest 30% of repair times.

$$P(x < k) = 0.30$$

$$P(x < k) = (\text{base})(\text{height}) = (k - 1.5) \cdot (0.4)$$

- **0.3 = (k − 1.5) (0.4)** ; Solve to find k:
- 0.75 = k − 1.5 , obtained by dividing both sides by 0.4
- **k = 2.25** , obtained by adding 1.5 to both sides

The 30th percentile of repair times is 2.25 hours. 30% of repair times are 2.5 hours or less.

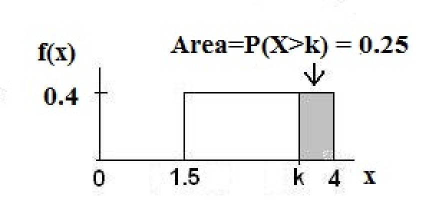
Exercise:

Problem:

The **longest 25%** of furnace repair times take **at least** how long? (Find the minimum time for the longest 25% of repairs.)

Solution:

Example 4 Figure 4



Uniform Distribution between 1.5 and 4 with an area of 0.25 shaded to the right representing the longest 25% of repair times.

$$P(x > k) = 0.25$$

$$P(x > k) = (\text{base})(\text{height}) = (4 - k) \cdot (0.4)$$

- **0.25 = (4 − k)(0.4)** ; Solve for k:

- 0.625 = 4 − k , obtained by dividing both sides by 0.4
- −3.375 = −k , obtained by subtracting 4 from both sides
- **k=3.375**

The longest 25% of furnace repairs take at least 3.375 hours (3.375 hours or longer).

Note: Since 25% of repair times are 3.375 hours or longer, that means that 75% of repair times are 3.375 hours or less. 3.375 hours is the **75th percentile** of furnace repair times.

Exercise:

Problem: Find the mean and standard deviation

Solution:

$\mu = \frac{a+b}{2}$ and $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

$\mu = \frac{1.5+4}{2} = 2.75$ hours and $\sigma = \sqrt{\frac{(4-1.5)^2}{12}} = 0.7217$ hours

Note: See "Summary of the Uniform and Exponential Probability Distributions" for a full summary.

**Example 5 contributed by Roberta Bloom

Glossary

Conditional Probability

The likelihood that an event will occur given that another event has already occurred.

Uniform Distribution

A continuous random variable (RV) that has equally likely outcomes over the domain, $a < x < b$. Often referred as the **Rectangular distribution** because the graph of the pdf has the form of a rectangle. Notation: X~$U(a,b)$. The mean is $\mu = \frac{a+b}{2}$ and the standard deviation is $\sigma = \sqrt{\frac{(b-a)^2}{12}}$ The probability density function is $f(x) = \frac{1}{b-a}$ for $a < x < b$ or $a \leq x \leq b$. The cumulative distribution is $P(X \leq x) = \frac{x-a}{b-a}$.

7.4 Continuous Random Variables: Practice TCC
In this module the student will explore the properties of data with a uniform distribution.

Student Learning Outcomes

- The student will analyze data following a uniform distribution.

Given

The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.

Describe the Data

Exercise:

Problem: What is being measured here?

Solution:

The age of cars in the staff parking lot

Exercise:

Problem: In words, define the Random Variable X.

Solution:

X = The age (in years) of cars in the staff parking lot

Exercise:

Problem: Are the data discrete or continuous?

Solution:

Continuous

Exercise:

Problem: The interval of values for x is:

Solution:

0.5 - 9.5

Exercise:

Problem: The distribution for X is:

Solution:

$X \sim U(0.5, 9.5)$

Probability Distribution

Exercise:

Problem: Write the probability density function.

Solution:

$f(x) = \frac{1}{9}$

Exercise:

Problem: Graph the probability distribution.

- **a**Sketch the graph of the probability distribution.

- **b**Identify the following values:
    - **i**Lowest value for x:
    - **ii**Highest value for x:
    - **iii**Height of the rectangle:
    - **iv**Label for x-axis (words):
    - **v**Label for y-axis (words):

Solution:

- **b.i**0.5
- **b.ii**9.5
- **b.iii** $\frac{1}{9}$
- **b.iv**Age of Cars
- **b.v** $f(x)$

Random Probability

Exercise:

Problem:

Find the probability that a randomly chosen car in the lot was less than 4 years old.

- **a**Sketch the graph. Shade the area of interest.

- **b**Find the probability. $P(x < 4)$ =

Solution:

- **b.** $\frac{3.5}{9}$

Exercise:

Problem:

Out of just the cars less than 7.5 years old, find the probability that a randomly chosen car in the lot was less than 4 years old.

- **a**Sketch the graph. Shade the area of interest.

- **b**Find the probability. $P(x < 4 \mid x < 7.5)$ =

Solution:

- **b** $\frac{3.5}{7}$

Exercise:

Discussion Question

Problem:

What has changed in the previous two problems that made the solutions different?

Quartiles

Exercise:

Problem: Find the average age of the cars in the lot.

Solution:

$\mu = 5$

Exercise:

Problem:

Find the third quartile of ages of cars in the lot. This means you will have to find the value such that $\frac{3}{4}$, or 75%, of the cars are at most (less than or equal to) that age.

- **a**Sketch the graph. Shade the area of interest.

- **b**Find the value k such that $P(x < k) = 0.75$.
- **c**The third quartile is:

Solution:

- **b**$k = 7.25$

7.5 Continuous Random Variables: Homework TCC

This module provides a number of homework exercises related to Continuous Random Variables.

For each probability and percentile problem, DRAW THE PICTURE!

Exercise:

Problem:

Consider the following experiment. You are one of 100 people enlisted to take part in a study to determine the percent of nurses in America with an R.N. (registered nurse) degree. You ask nurses if they have an R.N. degree. The nurses answer “yes” or “no.” You then calculate the percentage of nurses with an R.N. degree. You give that percentage to your supervisor.

- **a**What part of the experiment will yield discrete data?
- **b** What part of the experiment will yield continuous data?

Exercise:

Problem:

When age is rounded to the nearest year, do the data stay continuous, or do they become discrete? Why?

Exercise:

Problem:

Births are approximately uniformly distributed between the 52 weeks of the year. They can be said to follow a Uniform Distribution from 1 – 53 (spread of 52 weeks).

- **a** X~
- **b** Graph the probability distribution.
- **c** $f(x)$=
- **d** μ=
- **e** σ=

- **f** Find the probability that a person is born at the exact moment week 19 starts. That is, find $P(X = 19)$.
- **g** $P(2 < X < 31) =$
- **h** Find the probability that a person is born after week 40.
- **i** $P(12 < X \mid X < 28)$=
- **j** Find the 70th percentile.
- **k** Find the minimum for the upper quarter.

Solution:

- **a** X~$U(1{,}53)$
- **c** $f(x) = \frac{1}{52}$ where $1 \leq x \leq 53$
- **d** 27
- **e**15.01
- **f** 0
- **g** $\frac{29}{52}$
- **h** $\frac{13}{52}$
- **i** $\frac{16}{27}$
- **j** 37.4
- **k** 40

Exercise:

Problem:

A random number generator picks a number from 1 to 9 in a uniform manner.

- **a** X~
- **b** Graph the probability distribution.
- **c** $f(x) =$
- **d** $\mu =$
- **e** $\sigma =$
- **f** $P(3.5 < X < 7.25) =$
- **g** $P(X > 5.67) =$
- **h** $P(X > 5 \mid X > 3) =$

- **i** Find the 90th percentile.

Exercise:

Problem:

The speed of cars passing through the intersection of Blossom Hill Road and the Almaden Expressway varies from 10 to 35 mph and is uniformly distributed. None of the cars travel over 35 mph through the intersection.

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** What is the probability that the speed of a car is at most 30 mph?
- **h** What is the probability that the speed of a car is between 16 and 22 mph.
- **i** $P(20 < X < 53) =$ State this in a probability question (similar to **g** and **h**), draw the picture, and find the probability.
- **j** Find the 90th percentile. This means that 90% of the time, the speed is less than _____ mph while passing through the intersection per minute.
- **k** Find the 75th percentile. In a complete sentence, state what this means. (See **j**.)
- **l** Find the probability that the speed is more than 24 mph given (or knowing that) it is at least 15 mph.

Solution:

- **b** X~$U(10\ 35)$
- **d** $f(x) = \frac{1}{25}$ where $10 \leq X \leq 35$
- **e** $\frac{45}{2}$

- **f** 7.22
- **g** $\frac{4}{5}$
- **h** $\frac{6}{25}$
- **i** $\frac{3}{5}$
- **j** 32.5
- **k** 28.75
- **l** $\frac{11}{20}$

Exercise:

Problem:

According to a study by Dr. John McDougall of his live-in weight loss program at St. Helena Hospital, the people who follow his program lose between 6 and 15 pounds a month until they approach trim body weight. Let's suppose that the weight loss is uniformly distributed. We are interested in the weight loss of a randomly selected individual following the program for one month. (Source: **The McDougall Program for Maximum Weight Loss** by John A. McDougall, M.D.)

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** Find the probability that the individual lost more than 10 pounds in a month.
- **h** Suppose it is known that the individual lost more than 10 pounds in a month. Find the probability that he lost less than 12 pounds in the month.
- **i** $P(7 < X < 13 \mid X > 9) =$ State this in a probability question (similar to g and h), draw the picture, and find the probability.

Exercise:

Problem:

A subway train on the Red Line arrives every 8 minutes during rush hour. We are interested in the length of time a commuter must wait for a train to arrive. The time follows a uniform distribution.

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** Find the probability that the commuter waits less than one minute.
- **h** Find the probability that the commuter waits between three and four minutes.
- **i** 60% of commuters wait more than how long for the train? State this in a probability question (similar to **g** and **h**), draw the picture, and find the probability.

Solution:

- **b** X~$U(0,8)$
- **d** $f(x) = \frac{1}{8}$ where $0 \leq X \leq 8$
- **e** 4
- **f** 2.31
- **g** $\frac{1}{8}$
- **h** $\frac{1}{8}$
- **i** 3.2

Exercise:

Problem:

The age of a first grader on September 1 at Garden Elementary School is uniformly distributed from 5.8 to 6.8 years. We randomly select one first grader from the class.

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** Find the probability that she is over 6.5 years.
- **h** Find the probability that she is between 4 and 6 years.
- **i** Find the 70th percentile for the age of first graders on September 1 at Garden Elementary School.

Try these multiple choice problems

The next three questions refer to the following information. The Sky Train from the terminal to the rental car and long term parking center is supposed to arrive every 8 minutes. The waiting times for the train are known to follow a uniform distribution.

Exercise:

Problem: What is the average waiting time (in minutes)?

- **A** 0.0000
- **B** 2.0000
- **C** 3.0000
- **D** 4.0000

Solution:

D

Exercise:

Problem: Find the 30th percentile for the waiting times (in minutes).

- **A** 2.0000
- **B** 2.4000
- **C** 2.750
- **D** 3.000

Solution:

B

Exercise:

Problem:

The probability of waiting more than 7 minutes given a person has waited more than 4 minutes is?

- **A** 0.1250
- **B** 0.2500
- **C** 0.5000
- **D** 0.7500

Solution:

B

7.6 Continuous Random Variables: Review TCC
This module provides a number of homework/review problems related to Continuous Random Variables.

[link] – [link] refer to the following study: A recent study of mothers of junior high school children in Santa Clara County reported that 76% of the mothers are employed in paid positions. Of those mothers who are employed, 64% work full-time (over 35 hours per week), and 36% work part-time. However, out of all of the mothers in the population, 49% work full-time. The population under study is made up of mothers of junior high school children in Santa Clara County.

Let E =employed, Let F =full-time employment

Exercise:

Problem:

- **a** Find the percent of all mothers in the population that NOT employed.
- **b** Find the percent of mothers in the population that are employed part-time.

Solution:

- **a** 24%
- **b** 27%

Exercise:

Problem:

The type of employment is considered to be what type of data?

Solution:

Qualitative

Exercise:

Problem: In symbols, what does the 36% represent?

Solution:

$P(\text{PT} \mid E)$

Exercise:

Problem:

Find the probability that a randomly selected person from the population will be employed OR work full-time.

Solution:

0.7336

Exercise:

Problem:

Based upon the above information, are being employed AND working part-time:

- **a** mutually exclusive events? Why or why not?
- **b** independent events? Why or why not?

Solution:

- **a** No,
- **b** No,

[link] - [link] **refer to the following:** We randomly pick 10 mothers from the above population. We are interested in the number of the mothers that are employed. Let $X =$number of mothers that are employed.

Exercise:

Problem: State the distribution for X.

Solution:

$B(10\ 0.76)$

Exercise:

Problem: Find the probability that at least 6 are employed.

Solution:

0.9330

Exercise:

Problem:

A person invests $1000 in stock of a company that hopes to go public in 1 year.

- The probability that the person will lose all his money after 1 year (i.e. his stock will be worthless) is 35%.
- The probability that the person's stock will still have a value of $1000 after 1 year (i.e. no profit and no loss) is 60%.
- The probability that the person's stock will increase in value by $10,000 after 1 year (i.e. will be worth $11,000) is 5%.

Find the expected PROFIT after 1 year.

Solution:

$150

Exercise:

Problem:

Rachel’s piano cost $3000. The average cost for a piano is $4000 with a standard deviation of $2500. Becca’s guitar cost $550. The average cost for a guitar is $500 with a standard deviation of $200. Matt’s drums cost $600. The average cost for drums is $700 with a standard deviation of $100. Whose cost was lowest when compared to his or her own instrument? Justify your answer.

Solution:

Matt

Exercise:

Problem:

For the following data, which of the measures of central tendency would be the LEAST useful: mean, median, mode? Explain why. Which would be the MOST useful? Explain why.

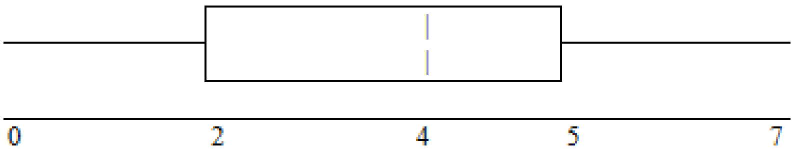
4,6,6,12 18 18 18 200

Solution:

Mean

Exercise:

Problem:



For each statement below, explain why each is either true or false.

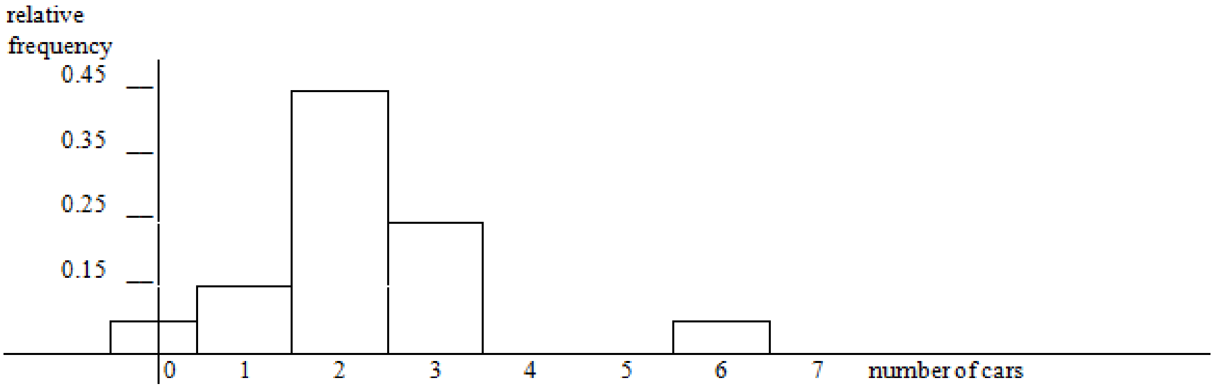
- **a** 25% of the data are at most 5.

- **b** There is the same amount of data from 4 – 5 as there is from 5 – 7.
- **c** There are no data values of 3.
- **d** 50% of the data are 4.

Solution:

- **a** False
- **b** True
- **c** False
- **d** False

[link] – [link] **refer to the following:** 64 faculty members were asked the number of cars they owned (including spouse and children's cars). The results are given in the following graph:



Exercise:

Problem: Find the approximate number of responses that were "3."

Solution:

16

Exercise:

Problem:

Find the first, second and third quartiles. Use them to construct a box plot of the data.

Solution:

2,2,3

[link] – [link] refer to the following study done of the Girls soccer team "Snow Leopards":

Hair Style		Hair Color	
	blond	brown	black
ponytail	3	2	5
plain	2	2	1

Suppose that one girl from the Snow Leopards is randomly selected.

Exercise:

Problem:

Find the probability that the girl has black hair GIVEN that she wears a ponytail.

Solution:

$\frac{5}{10} = 0.5$

Exercise:

Problem:

Find the probability that the girl wears her hair plain OR has brown hair.

Solution:

$\frac{7}{15}$

Exercise:

Problem:

Find the probability that the girl has blond hair AND that she wears her hair plain.

Solution:

$\frac{2}{15}$